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VOL. V. NO. 26. DECEMBER 17, 1908.

THE JOURNAL OF PHILOSOPHY
PSYCHOLOGY AND SCIENTIFIC METHODS

ON A SUPPOSED CRITERION OF THE ABSOLUTE TRUTH OF SOME PROPOSITIONS

PROFESSOR ROYCE in his address on "The Problem of Truth in the Light of Recent Research," delivered at the Third International Congress of Philosophy at Heidelberg, presents a criterion for the absolute truth of certain propositions which deserves our attention partly on account of the importance of the conclusions reached and partly on account of the peculiarity of the arguments used. After discussing three motives prevalent in recent investigations on the nature of truth, which are called instrumentalism, individualism, and absolute pragmatism, the author says:[1] "*Absolute truth is accessible to man in precisely so far as he can discover what it is that the will must necessarily do,* however capriciously and freely it acts, so long as it retains any kind of rational connection in its life. Now there is but one way to find this out, and that is to discover, unquestionably by experiments in the combination and construction of ideas, what forms of combination and construction there exist such that if you try to suppose them nonexistent, and if you nevertheless keep on acting at all, you inevitably contradict yourself. Exact mathematical research in the realm of the fundamental relations is able to show that the system of these relations has certain absolute forms. The only way to detect these forms is *by attempting to deny that they are present, or that they are what one first sees them to be.* In numerous cases (and these are precisely the cases where definite results can be reached) one hereupon discovers that *to deny certain propositions about these fundamental relations inevitably implies the reassertion of the very propositions denied.* Thus Euclid discovered that the form of the ordinal sequence of the whole numbers is such as to require that there should be no last prime number. This proposition relates to a highly abstract matter, namely, to the type of relationship which characterizes the ordinal sequence of the natural numbers. Euclid proved this proposition, however, by showing that if one asserted any prime to be the last prime number, one even thereby provided for oneself the means to construct a later prime number, so that the proposition: There is no last prime number, was shown to be a proposition whose contradictory implied the proposition itself which was to be proved. The method here used by Euclid is precisely the one upon whose elaboration, and repeated application, the most significant of the newer researches into the fundamental relations depend. The result is that one finds propositions such that the denial of them implies their truth. Such propositions are thereby shown to be true in precisely the sense which this address declares to be the genuine sense in which we may call truth absolute.'' Pure mathematics and modern pure logic contain absolute truths in great number.

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[1] The quotation is from the abstract which was prepared by the author for the use of the members of the congress; the italics are his.

The argumentation of the author is very clear: A proposition is absolutely true if its denial implies the proposition; this is exemplified by Euclid's proof of the theorem of the ''inexhaustibility of prime numbers.'' Theorems which are proved in this way are, indeed, very frequent in all parts of mathematics. The essential feature of the proof consists in showing that the opposite of the theorem in question involves a contradiction either with one of the principles or with a proposition proved and conceded previously, or with one of the conditions of the problem. In all these demonstrations one concludes, after having successfully shown that the opposite of the theorem in question is false, that the theorem must be true, since only one of two contradictory propositions can be true. These proofs are called apagogical, or indirect demonstrations in contradistinction to the direct demonstrations, in which the theorem is proved by showing how it can be deduced from the principles. Euclid begins his proof of the inexhaustibility of the prime numbers by assuming that this theorem is not true, *i. e.*, that there exists only a finite number of primes. In a finite series of quantities, all of which are different, one term must be greater than all the other terms, and it follows from this that there must be a prime number which is the greatest of all. This consequence, however, implies the contradiction that one can readily find at least one number which is greater than the number that was supposed to be the last prime and which can not be divided by it or by any one of the smaller prime numbers.

The author attributes the character of absolute truth only to those mathematical propositions proved indirectly. It is very natural to ask whether there exists such a fundamental difference between the theorems proved directly and those proved indirectly as would authorize us to attribute the character of absolute truth to one group of theorems and not to the other. Direct and indirect demonstrations, as a matter of fact, are used promiscuously for the proof of any theorem, except, of course, in cases where the difficulties of the demonstration are so great that only one method of demonstrating could be found until now. This is, however, the case with only very complicated theorems, the truth of which depends so obviously on that of simpler propositions that one certainly will not attribute the character of absolute truth to the complicated propositions if it does not belong to the simpler ones. If a direct proof is given of a proposition which was proved formerly by the indirect method, does it lose its character of absolute truth? This is a pertinent question, since it refers to the very example given by the author. There is not the slightest difficulty in demonstrating the theorem of the inexhaustibility of prime numbers directly, since Dirichlet has discovered the following theorem: An infinite arithmetical series, the first term of which and the difference between two consecutive terms of which are prime numbers, contains an infinity of primes. Euclid's proposition follows from Dirichlet's theorem by direct proof, since the ordinal sequence of the whole numbers is a series such as spoken of in Dirichlet's theorem. It is not always easy to find a direct demonstration of a theorem which is proved apagogically, but it is always a very simple matter to find an indirect demonstration of a theorem which was demonstrated directly. All that is needed for this purpose is to deduce from the opposite of the theorem in question a conclusion which contradicts any one of the propositions constituting the *nervus probandi* of the direct proof. If we give an indirect demonstration of a proposition which was proved directly formerly, do we create an absolute truth by this apagogical demonstration? The author's distinction could be sustained if there were some theorems which can not be proved directly and others which can not be demonstrated indirectly, but it is contradicted by the fact that the two types of demonstrations are used promiscuously.[2]

It is not difficult to see that the difference between direct and indirect demonstration can not possibly constitute a thoroughgoing difference between mathematical propositions. A system of mathematics is the class of all propositions which can be deduced by merely logical processes from a given set of fundamental propositions. From this it follows that the opposite of any such theorem does not belong to this class, *i. e.,* that it is in contradiction to the principles. To show that a certain proposition involves a contradiction, or to show that its opposite can be obtained from the set of fundamental principles by deductions involving merely logical processes, is the same. From this it follows that every theorem belonging to a system of mathematics can be demonstrated by apagogical proof. For this reason one can not attribute the character of absolute truth to certain mathematical theorems because they were proved indirectly: absolute truth belongs to all mathematical propositions or to none of them.

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[2] The remark that the method of demonstration can not involve a fundamental difference between mathematical propositions, because every theorem which is proved directly can be proved indirectly, was also made by Professor Jerusalem in the course of the discussion following the author's address.

The fact that the indirect demonstration of a theorem does not give it prominence among the other theorems belonging to the same system becomes apparent in the case of conclusions drawn from sets of principles which are obviously arbitrary. It is not quite obvious to everybody not trained in modern pure logic that the choice of the set of fundamental propositions of mathematics is more or less arbitrary, as this is best seen from the discussions of non-Euclidean geometry. This difficulty is still greater in the case of algebra, and for this reason the following example was devised. It refers to a set of principles about the logical arbitrariness of which there is not the slightest doubt. Let the set of fundamental propositions be represented by the rules and regulations of the U. S. mail service regarding the acceptance and forwarding of domestic money orders. A man *A* owes to *B* the sum of $5.06. *B* moves to another town, and asks *A* to send all the money by one money order and to subtract at once the cost of the money order from the original debt. In this case the proposition holds that *A* can not comply with *B's* instructions. Let us start from the assumption that our proposition is false, *i. e.*, that *A* can send the money in the way required by *B*. If the money is to be sent by one money order, *A* must send it either at the rate for orders from $2.51 to $5, which is 5 cents, or at the rate of 8 cents for orders from $5.01 to $10. In the first case *A* would have to subtract 5 cents from $5.06, which leaves $5.01, which is accepted only at the rate of 8 cents; but if *A* tries to send the money at the higher rate, he has left only $4.98, which is forwarded at the rate of 5 cents. In the first case *A* is 3 cents short, and in the second case he has 3 cents left. The very supposition, therefore, that *A* can send the money as instructed by *B* implies that he can not do it. The proposition is proved indirectly and the author's criterion for absolute truth is obviously fulfilled, but who will be inclined to call this proposition alsolutely true?

Let us see in which sense mathematical propositions may be called true. It may be allowed to follow the presentation of the problem recently given in connection with an inquiry into some of the problems of psychophysics.[3] Mathematical propositions are derived from a set of fundamental propositions by merely logical processes; these propositions should be necessary and sufficient, which implies that they are independent of each other. The logical truth of a proposition consists in its agreement with these principles, but these principles themselves are logically neither true nor false, because each one of them may be substituted by its logical opposite without contradiction. The logical truth of a proposition refers to a certain set of fundamental principles, and if a proposition is true in regard to one set it certainly will not be true in regard to another set obtained from the first by changing one of the principles necessary for deducing the proposition. From this it follows that the truth of every mathematical theorem depends on the truth of the set of fundamental propositions from which it is derived, and no theorem can be said to be absolutely true since every single proposition, or every group of propositions, or the whole set of propositions from which it is deduced, may be denied. No mathematical proposition possesses absolute truth, since there exists no logical necessity for accepting any system of fundamental propositions. The author's criterion of the absolute truth of a proposition, therefore, can be sustained neither in particular when restricted to propositions which are proved indirectly nor in general when referred to mathematical propositions of every description.

Designating by the letter *N* a certain proposition, and by *M* the set of fundamental propositions which are necessary and sufficient to deduce this proposition, then the logical truth of it means merely that "*M* implies *N*." This proposition is independent of the acceptance or denial of the fundamental propositions, since the statement "*M* implies *N*" refers merely to the logical connection between *M* and *N* and contains nothing more than the assertion of the fact that *N* is a logical consequence of *M*. Let us apply these results to the two examples given above. We designate by *M* in the first case the set of propositions necessary and sufficient to deduce the system of ordinary algebra, and by *N* the theorem of the inexhaustibility of prime numbers. The truth of the proposition *N* depends entirely on the acceptance or denial of *M*, but the truth of the proposition "*M* implies *N*" is independent of *M*. Designating in a similar way by *M* the rules and regulations for the acceptance of money orders by the U. S. post offices, and by *N* the proposition which states the impossibility of sending the money as required by *B*, we obtain again the proposition "*M* implies *N*," which is independent of the truth of

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[3] F. M. Urban, "The Application of Statistical Methods to the Problems of Psychophysics," 1908, pp. 139–179, especially pp. 142–147.

M. The entire system of forwarding money may be changed, but it will remain true that under the given conditions *A* is unable to carry out *B's* instructions. An infinite number of statements of the form "*M* implies *N*" may be made, since *M* may represent any system of fundamental propositions, and *N* any proposition deduced from them, but it seems that the logical process involved is always the same. The fundamental principles of a system of thought state the general qualities of the objects treated of in this system. A single step in a demonstration consists always in showing that a certain object or a certain operation is such as is stated in one of the fundamental principles, from which one concludes that the object must have certain qualities or that the operation is allowable. The conclusion proceeds from the general to the particular. Propositions of the type "*M* implies *N*" contain the assertion of the correct performance of a subsumption, or a chain of subsumptions, a process which implies that there does not exist a logical contradiction between the singular and the general under which it is classified. These propositions do not deserve the name of special truths, since they state merely the requirement to which every proposition must conform in order to be an element of a given system of science.

Abstract systems of science have nothing to do with the existence or non-existence of the objects they speak of, and they become applicable to a certain realm of experience only by the statement that its objects are such as are spoken of in the system applied. Such a statement must be founded on experience, and one tries in vain to deduce the existence of any object from a group of abstract propositions. A proposition is called empirically true if it contains the description of an empirical fact, and a whole system of science is called empirically true if an empirical fact corresponds to every proposition of the system. One may be sure that all the propositions of a system are empirically true if they are deduced from a set of fundamental propositions that are empirically true. In this respect the principle holds that a proposition deduced by merely logical processes from a set of propositions which possess empirical truth is empirically true. Logical truth and empirical truth coincide if the set of fundamental propositions possesses empirical truth.

It is very difficult to make sure of the empirical truth of the propositions which form the basis of those systems of thought which are called empirical sciences in the restricted sense of the word, but it is very easy in those cases where the system of principles is arbitrary. Such cases are very serviceable as illustrations of the difference between empirical and logical truth. There is an infinity of systems imaginable for the acceptance of money orders, all of which are logically true, but among these systems there is only one which has empirical truth too, namely, the one which is actually in force and which is represented by those rules and regulations with which one has to comply if one wishes to forward money through the U. S. postal service. Every logical conclusion drawn from this set is also empirically true, and the system is complete, since the order will be accepted only in one of the cases provided for by the law. The proposition "*M* implies *N*" will remain true and the character of logical truth of the proposition *N* will be preserved, even if an entirely new set of regulations for the acceptance of money orders should be passed. Our problem will thus retain its humoristic value as a conundrum, which it shares with many a problem of modern pure logic, but the proposition *N* will cease to be empirically true when our man *A* will be able to forward the money as required by *B*.

To attribute the character of absolute truth to the proposition "*M* implies *N*" is equivalent to saying that the principles of logic are absolutely true. To regard every single proposition of this form as an absolute truth increases our stock of absolute truths enormously, though in a somewhat doubtful way, since propositions of this type can be formed not only of the theorems of mathematics, but also of any classification or definition no matter how arbitrary it may be. Let a definition be given in the form $A = abc \ldots$ then each one of the propositions "$A = abc \ldots$ implies A is a;" "$A = abc \ldots$ implies A is b," . . . must be called absolutely true. The definition may be perfectly arbitrary, but the truth of the implication does not depend on the acceptance or denial of it. In a similar way let us designate by the letter *M* the lemma "*A* is either *a*, or *b*, . . . or *n*," then each one of the propositions "*M* implies, *a* is *A*," "*M* implies, *b* is *A*," . . . is a proposition the truth of which does not depend on the acceptance of the lemma. These examples show clearly that the formal character of all these propositions is the same and that their truth is identical with that of the principles of logic.

The principles of logic are, therefore, the only propositions which regard abstract systems of thought and the truth of which is independent of any supposition. They may be said to be absolutely true in so far as no proposition of an abstract system must contradict them, but there remains the problem why phenomena of actual experience also must comply with them. This is required by the above-mentioned principle that a proposition deduced by merely logical processes from a set of empirically true propositions must be empirically true. We may express it in short by asking how the principles of logic come to have empirical truth. This question refers to the problem why the laws of thought are also the laws of things, a problem the solution of which is begged if one declares the principles of logic to be absolutely true.

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THE TRUE GOD OF SCHOLASTICISM

IN the celebrated work on "Pragmatism" of our distinguished psychologist and philosopher, William James, we read the following lines: "Old fashioned theism was bad enough, with its notion of God as an exalted monarch, made up of a lot of unintelligible or preposterous attributes; but so long as it held strongly by the argument from design, it kept some touch with concrete realities. Since, however, darwinism has once for all displaced design from the minds of the 'scientific,' theism has lost that foothold; and some kind of an immanent or pantheistic deity working in things rather than above them is, if any, the kind recommended to our contemporary imagination. Aspirants to a philosophic religion turn, as a rule, more hopefully nowadays towards idealistic pantheism than towards the other dualistic theism, in spite of the fact that the latter still counts able defenders" (p. 70).

Speaking about traditional theism, Professor James had already said: "It is the lineal descendant, through one stage of concession after another, of the dogmatic scholastic theism still taught rigorously in the seminaries of the Catholic church. For a long time it used to be called among us the philosophy of the Scottish school" (pp. 17, 18).

And a few lines further he gives the following description of the "theistic God": "You have to go to the world which he has created to get any inkling of his actual character: he is the kind of God that has once for all made that kind of a world. The God of the theistic writers lives on as purely abstract heights as does the absolute. Absolutism has a certain sweep and dash about it, while the usual theism is more insipid, but both are equally remote and vacuous" (pp. 19, 20).

On reading these lines, the student of post-Kantian philosophy is at once reminded of Hegel's sharp attack on the God of old metaphysics. Like William James to-day, the author of the "Logik" had told us of a metaphysical system in which "the terms of thought were cut off from their connection, their solidarity"; of a system in which a knowledge of the absolute was supposed to be gained by